Wöhler or Paris?

An old question revisited

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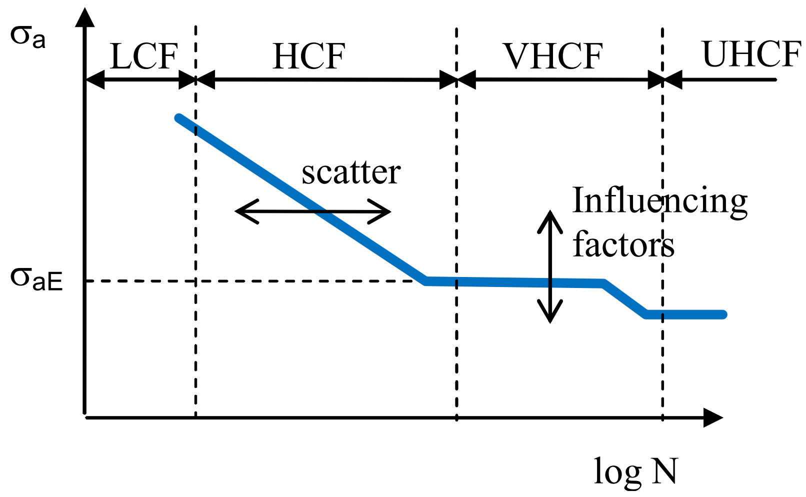
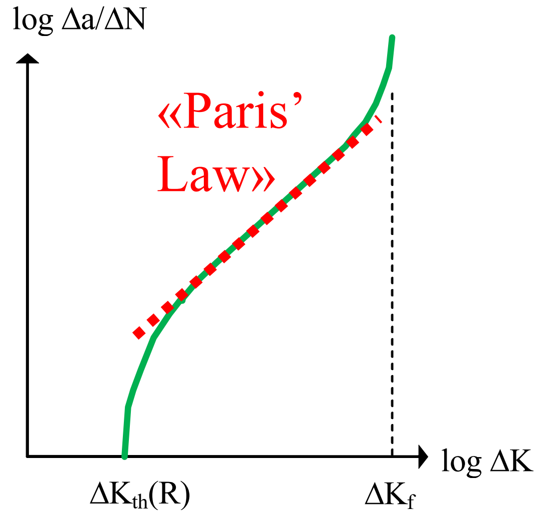
The «Old Question»

«Classical» fatigue or Fracture Mechanics?



S-N-curve («Wöhler-curve»)



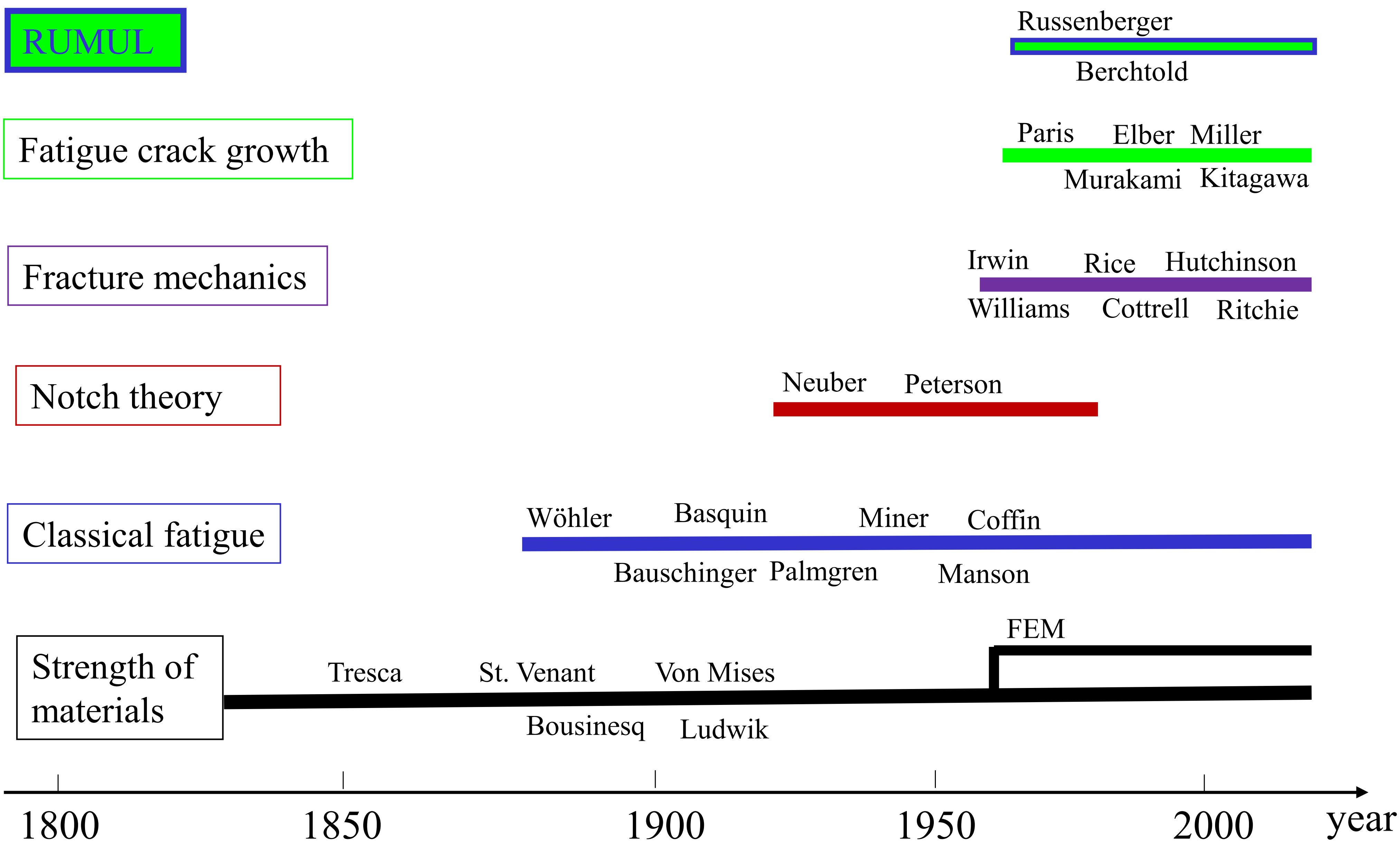
«da/dN-curve»

Drawbacks of the Wöhler-Curve:

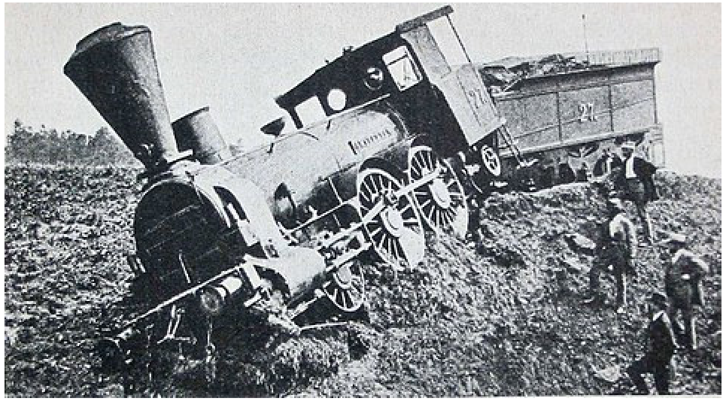
- Large scatter in N
- Numerous influencing factors (mean stress, size, shape, notch radii, surface treatment, mech. properties, environment, residual stress, etc.)
- No distinction between crack initiation, crack growth and fracture

WERKSTOFF-MECHANIK

History of fatigue of materials



Progress of science driven by catastrophic failures



Broken locomotive axle
Timelkam (Oe) (1875)

Collapse of steel bridge,
Münchenstein (1888)

Brittle fracture of welded
structures (Liberty ships (1940-45)

↓ Fatigue of metals — Wöhler

↓ Elastic-plastic buckling — Tetmajer

↓ Fracture mechanics — Irwin

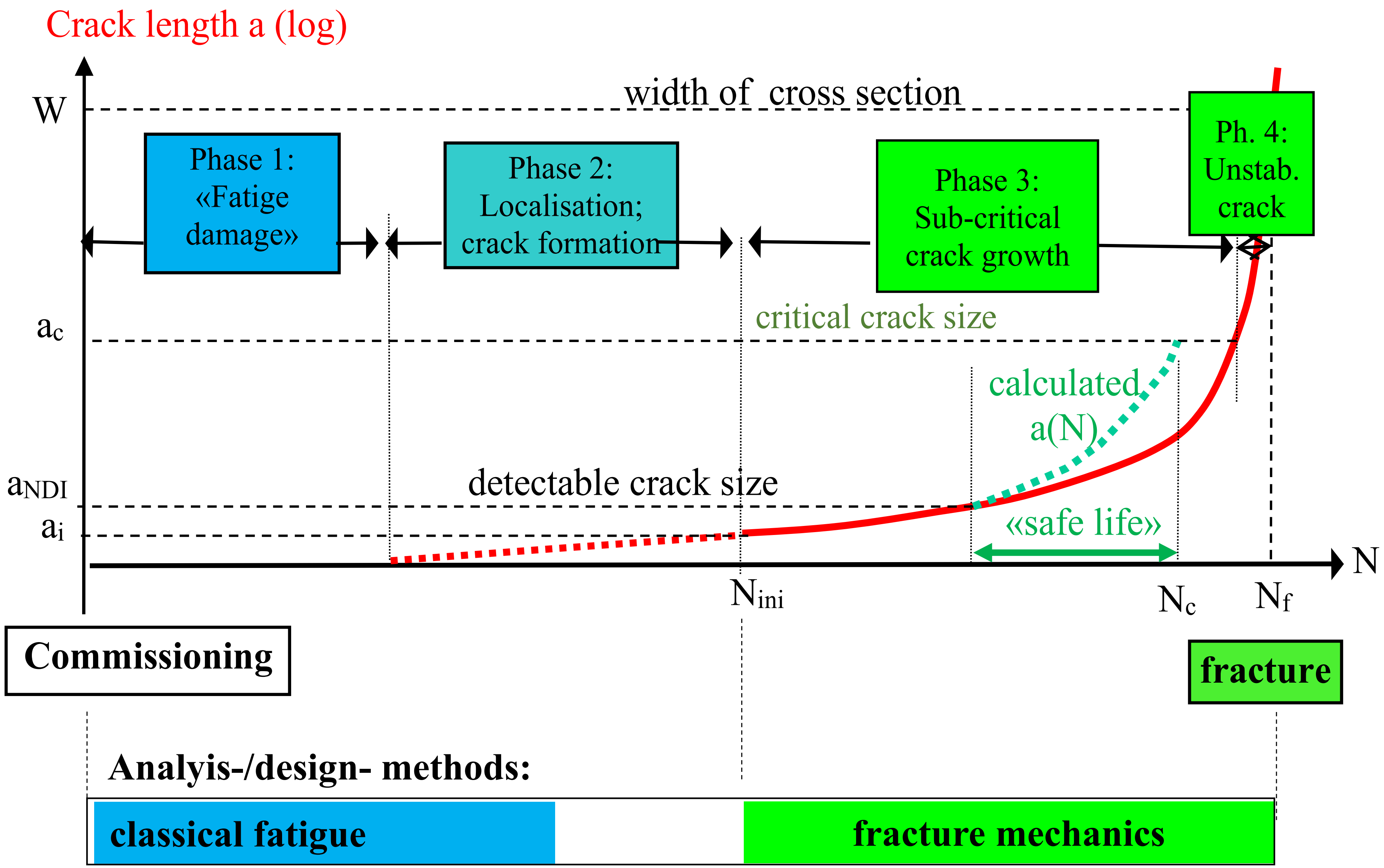
Underlying problems still not fully solved …

Switzerland, 2023

Germany, 2016

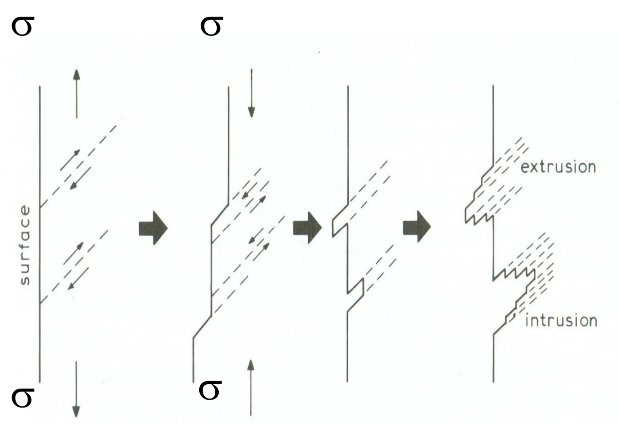
Suez, 2005

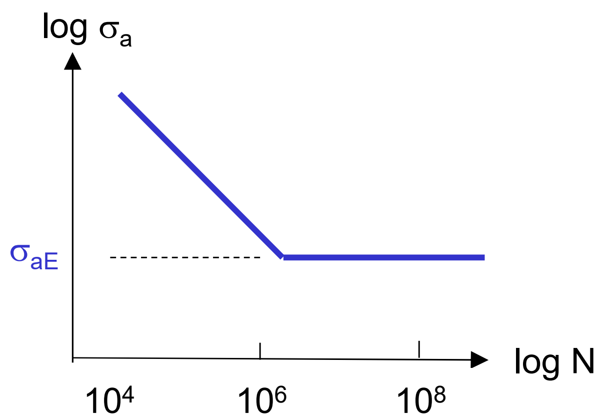
Process of a fatigue fracture



Mechanisms and theoretical approaches

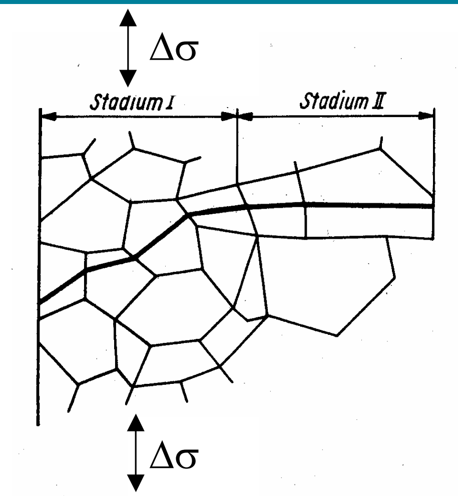
Phase 1: Fatige damage





Classical fatigue

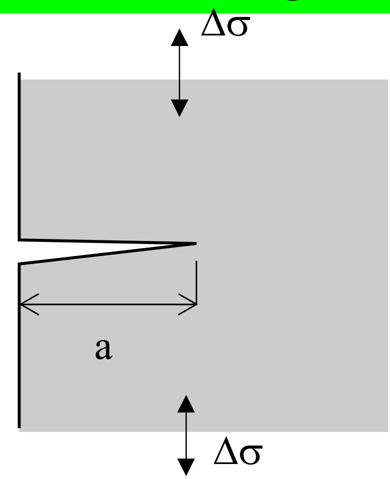
Phase 2: Crack formation

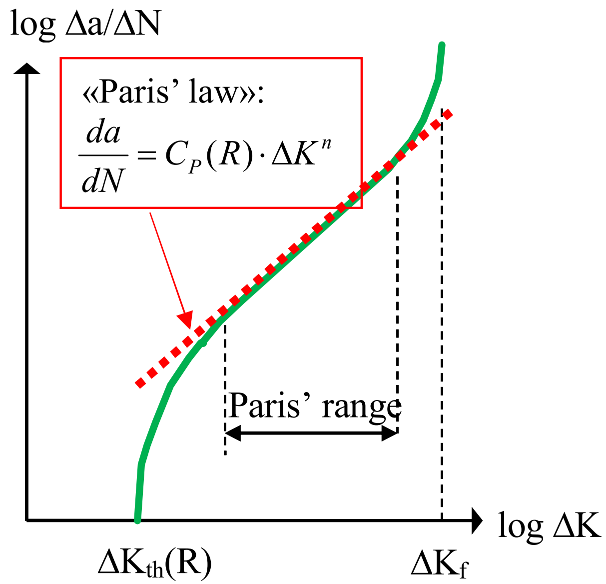


- Kitagawa-Diagramm
- El-Haddad a_0
- Fatigue R-curve
- Fatigue FAD

Elastic-plastic FM (EPFM)

Phase 3: Crack growth

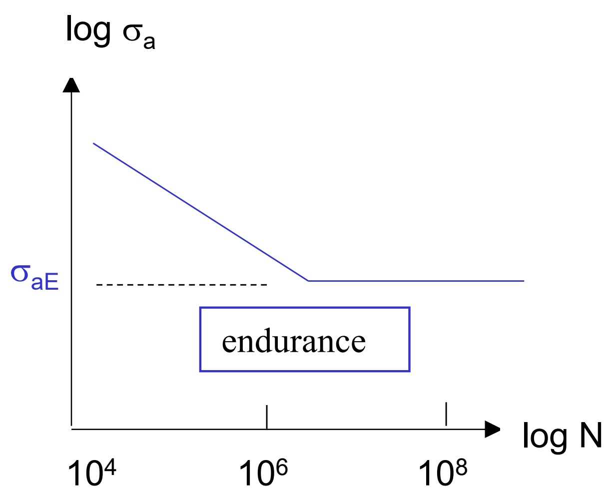




Linear-elastic FM (LEFM)

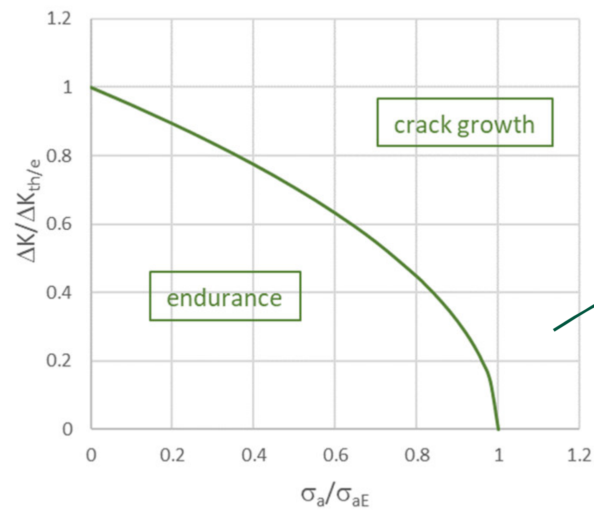
Endurance

«No» crack



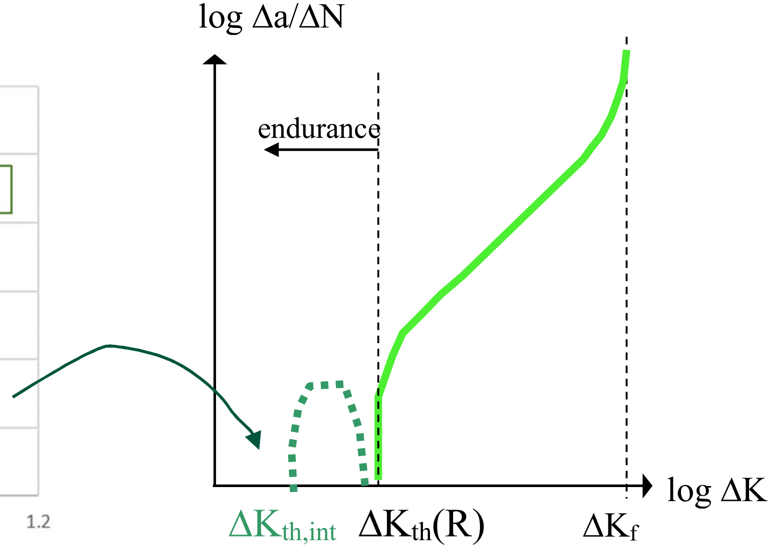
«Short» crack

a < ca. 1 mm



«Long» crack»

a > ca. 1 mm



Conditions for Endurance:

$$\sigma_a \leq \sigma_{aE}(\sigma_m)$$

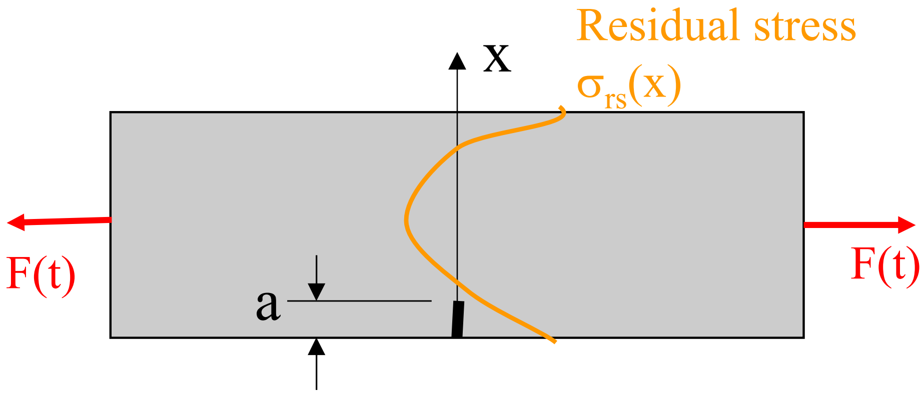
$$\frac{\Delta K}{\Delta K_{th}} = \sqrt{1 - \frac{\sigma_a}{\sigma_{aE}}}$$

«Fatigue FAD»

$$\Delta K \leq \Delta K_{th}(R)$$

maTec WERKSTOFF-MECHANIK

Safe Life Calculation based on Paris' Law



$$\frac{da}{dN} = C_P(R) \cdot \Delta K^n \quad \text{for } \Delta K > \Delta K_{th}(R)$$

$$\frac{da}{dN} = 0 \quad \text{for } \Delta K < \Delta K_{th}(R)$$

K_I due to residual stress:

$$K_{rs}(a) = \int_0^a \sigma_{rs}(x) \cdot h(x,a) \cdot dx$$

$$\Delta K = K_I(F_{\max}) - K_I(F_{\min})$$

$$R(a) = \frac{K_{\min}}{K_{\max}} = \frac{K_I(F_{\min}) + K_{rs}(a)}{K_I(F_{\max}) + K_{rs}(a)}$$

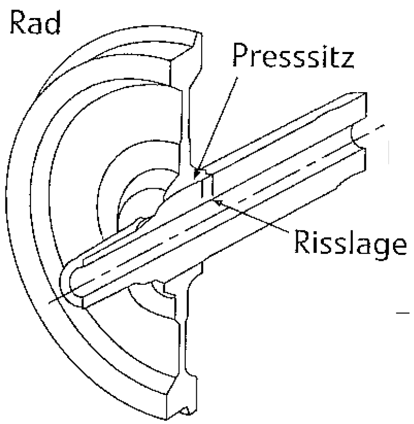
Safe Life: $$N_c(a) = \int_{a_0}^{a_c} \frac{da}{C_P(R(a)) \cdot \Delta K^n(a)}$$

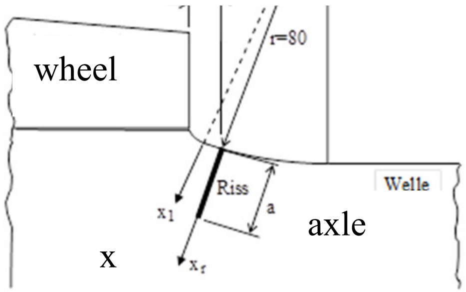
a_0: initial crack length ; e.g. $a_0 = a_{NDI}$

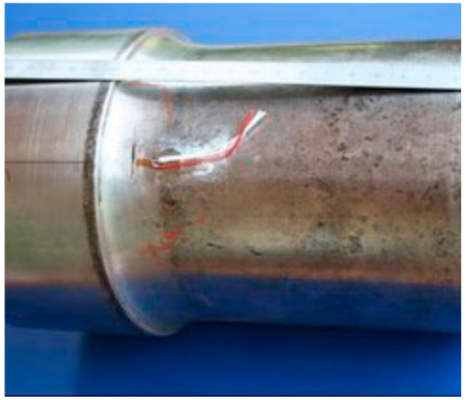
a_c: critical crack length

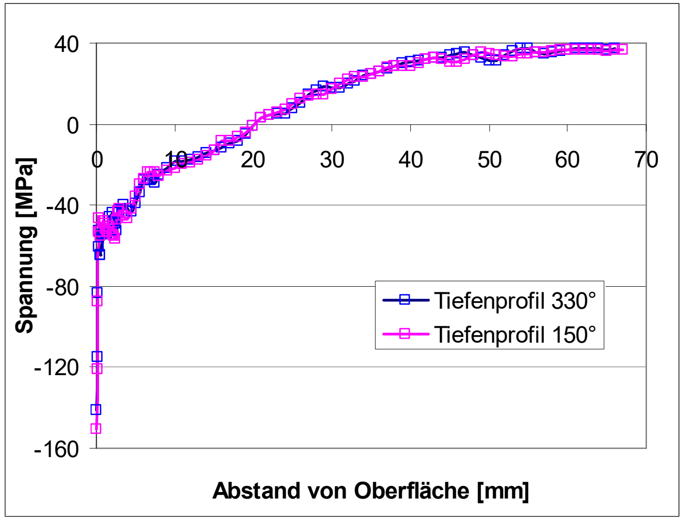
N_F highly sensitive to a_0, ΔK_{th} and residual stresses

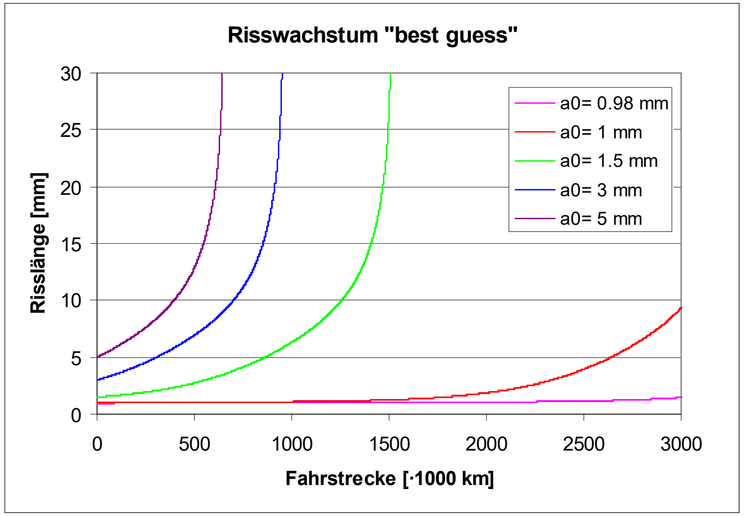
Example: Effect of a_0 and residual stresses on «safe life» of a railway axle

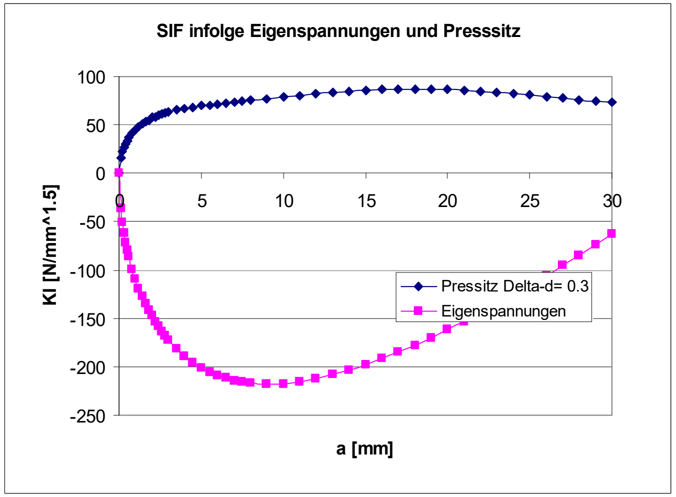


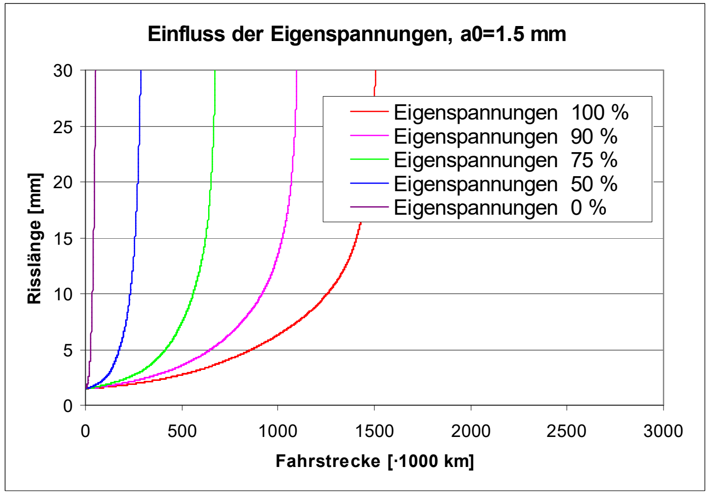












Drawbacks of „Paris‘ law“

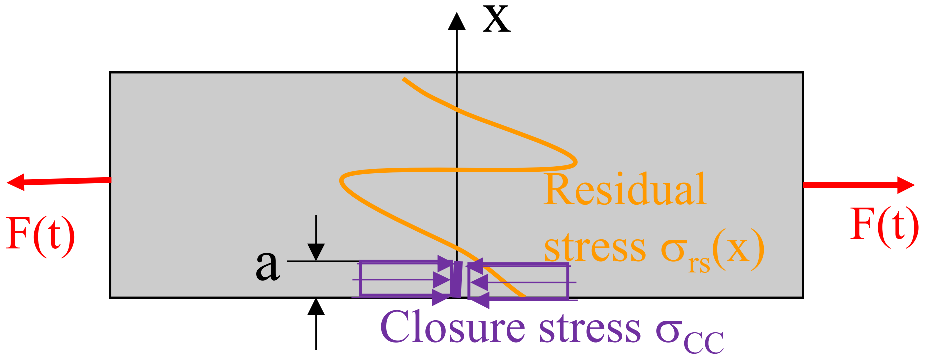
- Purely empirical correlation between da/dN and ΔK.
- Strange, intransparent dimension of the «Paris-Constant» C_p
- «C_p and n not material constants, but dependent on various parameters, such as R-ratio, T-stress, yield stress, specimen thickness, constraints, temperature, load history, loading range, etc.
- Limited, but unknown range of applicability
- Large scatter in threshold data

→ **Major improvements achievable by using analytical models as far as possible**

Main issues to be considered

- Effective vs. nominal crack load
- Analytical enrichment of Paris' Law
- Analytical enrichment of threshold values
- Residual stresses to be taken into account
- Including Phase 2 in safe life prediction
  (fatigue R-curve, fatigue FAD)

Effective fatigue crack load



K_I required to overcome crack closure:

$K_{op} = \max((K_{DICC} + K_{CICC}); K_{PICC})$

Damage-induced CC, Corrosion-induced CC, plasticity-induced CC

Nominal fatigue load:

$$\Delta K = K_I(F_{\max}) - K_I(F_{\min})$$

$$R(a) = \frac{K_{\min}}{K_{\max}} = \frac{K_I(F_{\min}) + K_{rs}(a)}{K_I(F_{\max}) + K_{rs}(a)}$$

$$K_{rs}(a) = \int_0^a \sigma_{rs}(x) \cdot h(x,a) \cdot dx$$

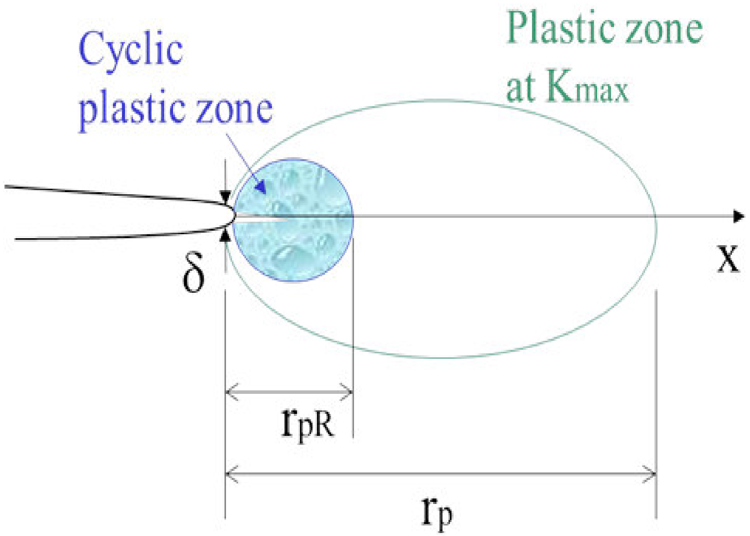
«Effective» fatigue load:

$$\Delta K_{eff} = K_I(F_{\max}) + K_{rs}(a) - K_{\min,eff}$$

$$R_{eff}(a) = \frac{K_{\min,eff} + K_{rs}(a)}{K_I(F_{\max}) + K_{rs}(a)}$$

$$K_{\min,eff} = \max(K_{op}; K_I(F_{\min}) + K_{rs})$$

Theoretical confirmation and refinement of Paris' Law



$$r_p = \frac{K_{max}^2}{2\pi \cdot (m \cdot \sigma_f)^2} \qquad r_{pR} = \frac{\Delta K^2}{4\pi \cdot (m \cdot \sigma_{UTS})^2}$$

$$\Delta\delta \cdot N_f^q = \delta_i - \delta_{max} \qquad \Delta\delta = \frac{K_{max}^2 - K_{min}^2}{2E \cdot m \cdot R_p}$$

$$N_f = \left[\frac{\delta_i - \delta_{max}}{\Delta\delta}\right]^{1/q} = \left[\frac{\delta_i - \delta_{max}}{\delta_{max} - \delta_{min}}\right]^{1/q}$$

$$\frac{da}{dN} = C_{nd} \cdot \frac{0.088^{\frac{n-2}{2}}}{157 \cdot \sigma_{UTS}^2 \cdot K_i^{n-2}} \cdot \left[\frac{1 - R_{eff}^2}{(1 - R_{eff})^2}\right]^{\frac{n-2}{2}} \cdot \Delta K_{eff}^n \quad \text{for } R_{eff} < 0.7$$

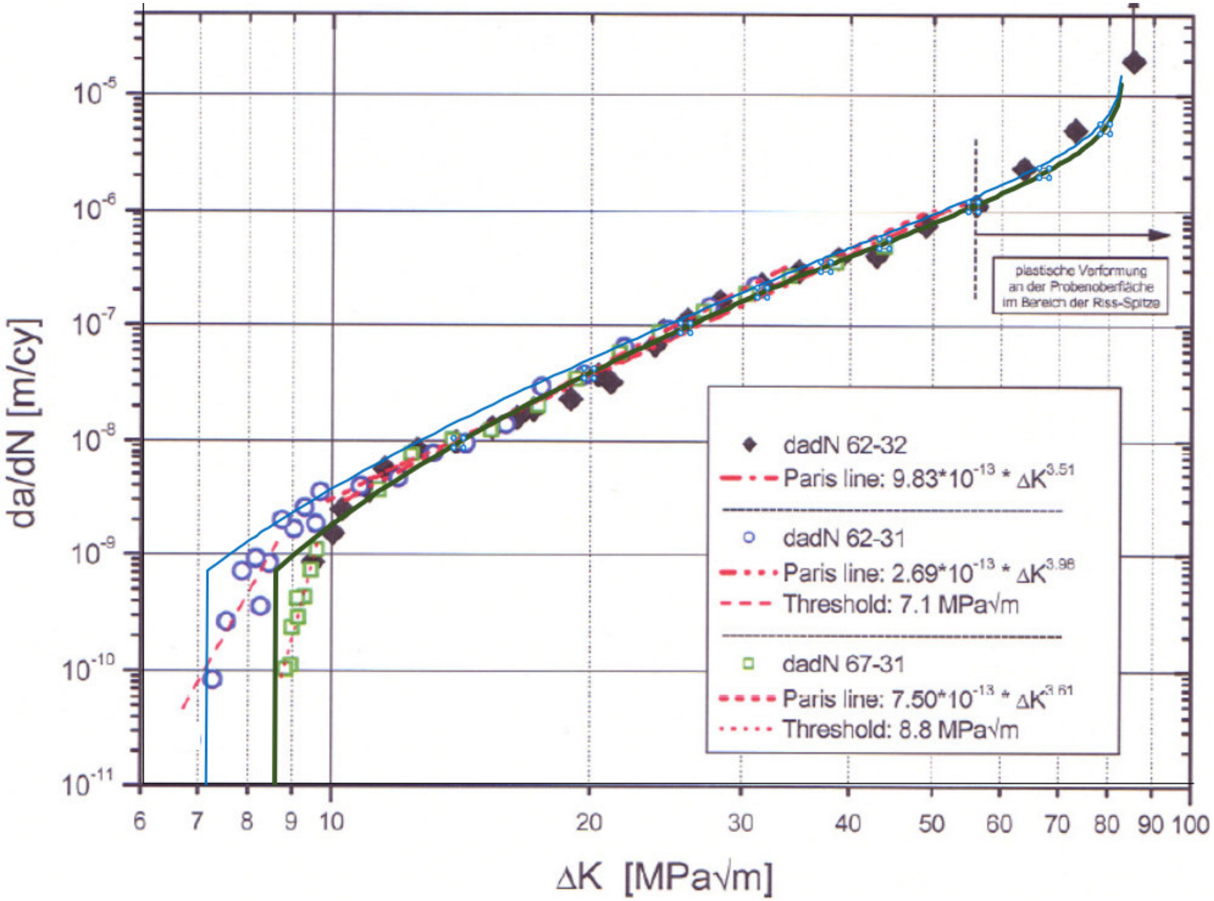
$$\frac{da}{dN} = C_{nd} \cdot \frac{0.5^{\frac{n-2}{2}}}{157 \cdot \sigma_{UTS}^2 \cdot K_i^{n-2}} \cdot \Delta K_{eff}^n \quad \text{for } R_{eff} > 0.7$$

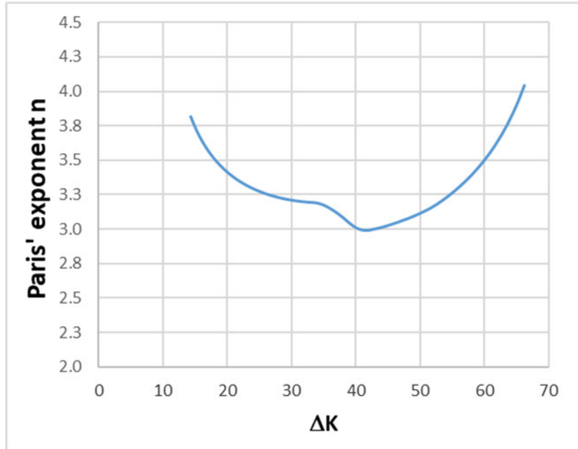
C_{nd} , n: Nondimensional open constants

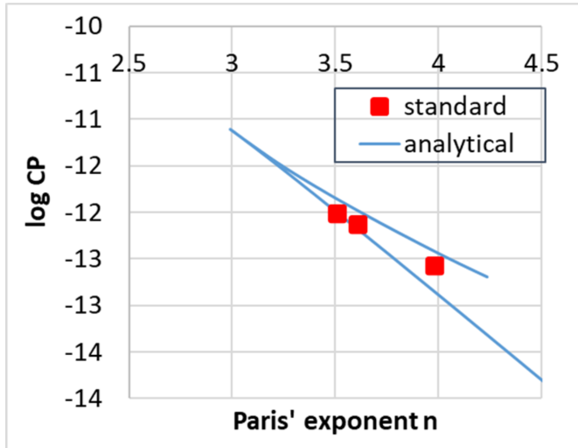
σ_{UTS} : tensile strength $\qquad$ K_i : Fracture toughness

Fitting to experimetal da/dN-curve

Example: Steel C35 (railway car axle)



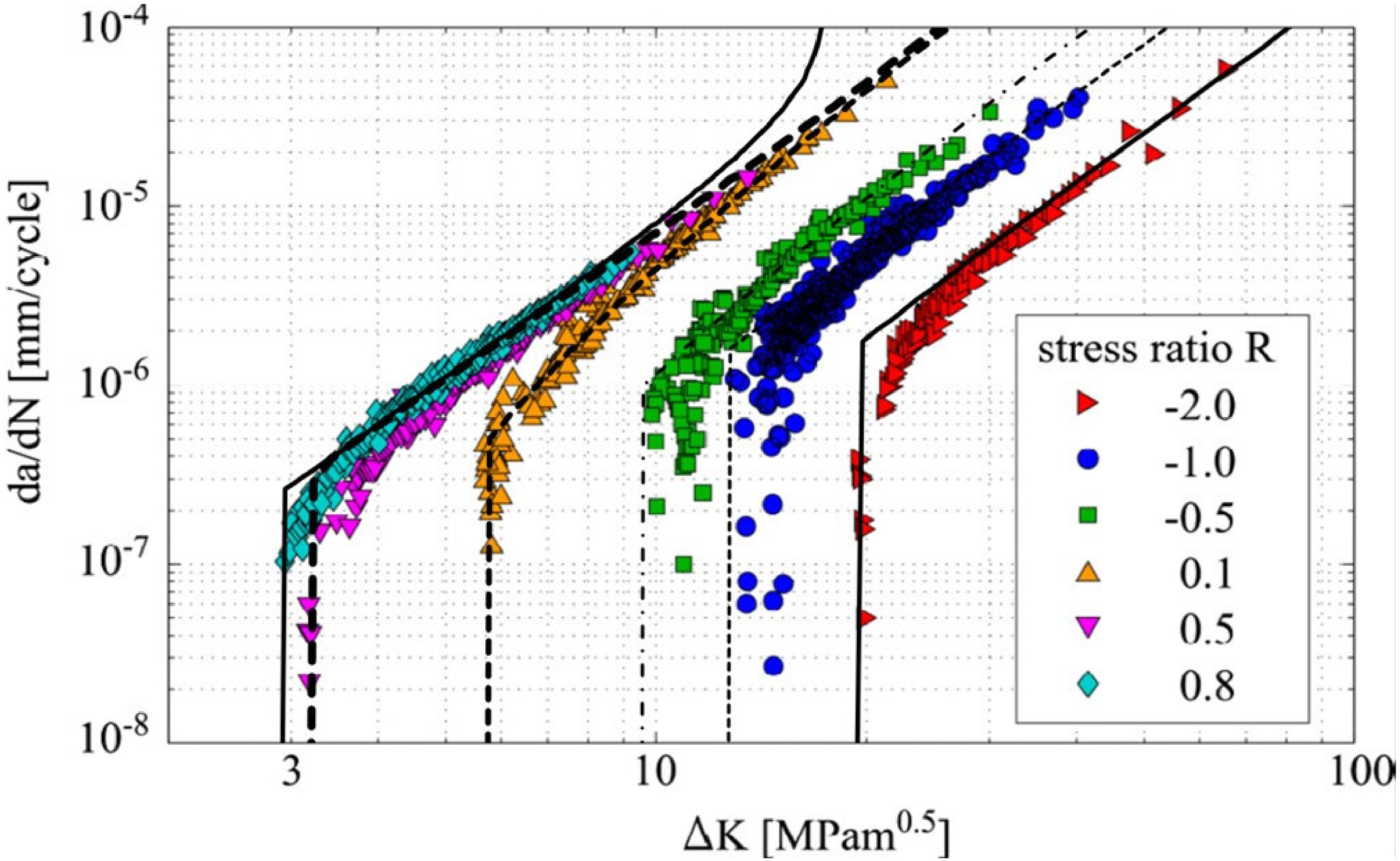




Determined open parameters from test 62-31:

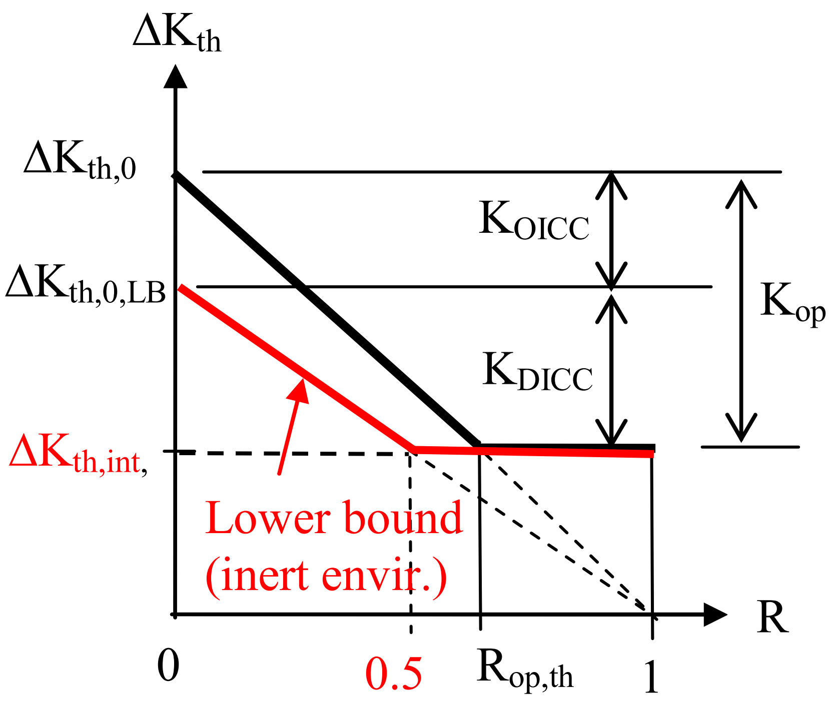
$C_{nd} \approx 0.05$; $n = 2.7$; $\Delta K_{th0} = 8.0$ $MPam^{0.5}$; $K_i = 92$ $MPam^{0.5}$; $K_{OICC} = 2.7$ $MPam^{0.5}$; $K_{PICC} = 0$

Analytical prediction of da/dN-curve of another steel for different R



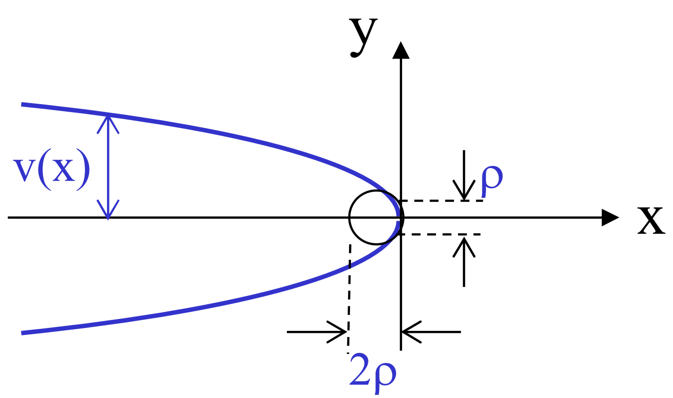
Railway axle steel EA4T

Analytical view on ΔK_{th}



$\Delta K_{th} = \Delta K_{th,0} \cdot (1-R)$	*for* $R < R_{op,th}$
$\Delta K_{th} = \Delta K_{th,int}$	*for* $R > R_{op,th}$

Estimation of intrinsic threshold:



$$v(x) = \frac{4 \cdot K_I}{E \cdot \sqrt{2\pi}} \cdot \sqrt{-x} \quad \Rightarrow \quad \rho = \frac{4K_I^2}{\pi \cdot E^2}$$

Condition for emission of dislocatîons:

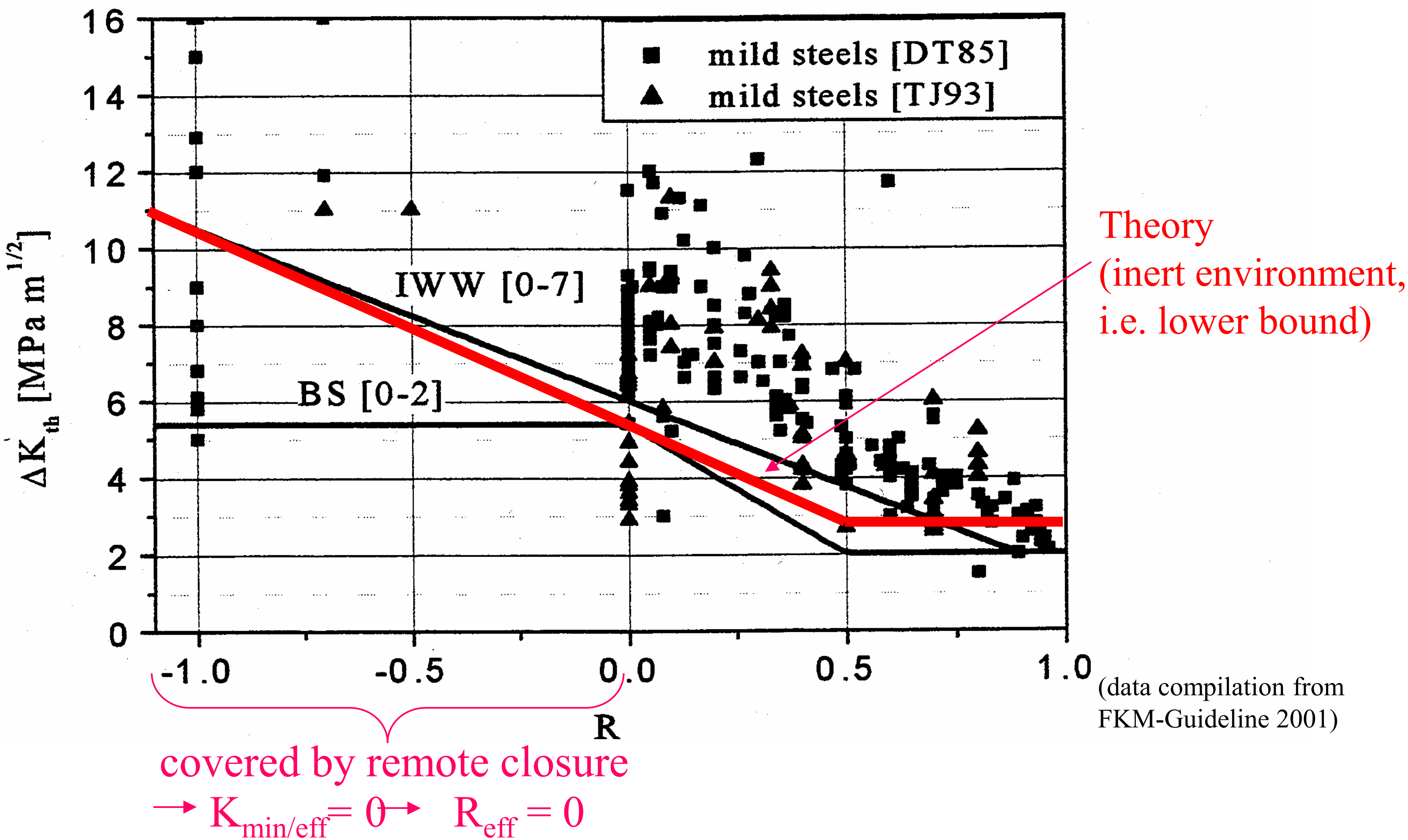
$\rho > b$ (b: Burgers Vektor)

$$\Rightarrow \Delta K_{th,\text{int}} = \frac{\sqrt{\pi}}{2} \cdot E \cdot \sqrt{b}$$

Lower bound for $\Delta K_{th,0}$:

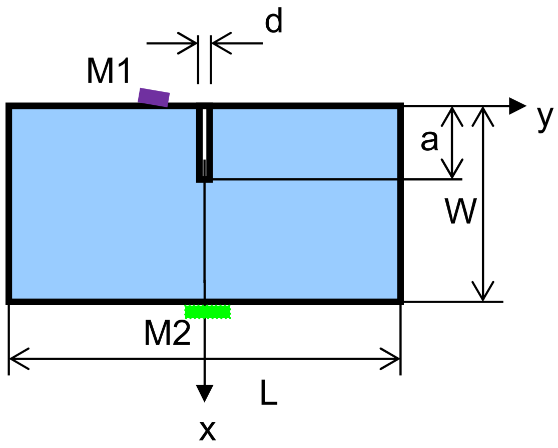
$$\Delta K_{th,0,LB} = 2 \cdot \Delta K_{th,\text{int}} = E \cdot \sqrt{\pi \cdot b}$$

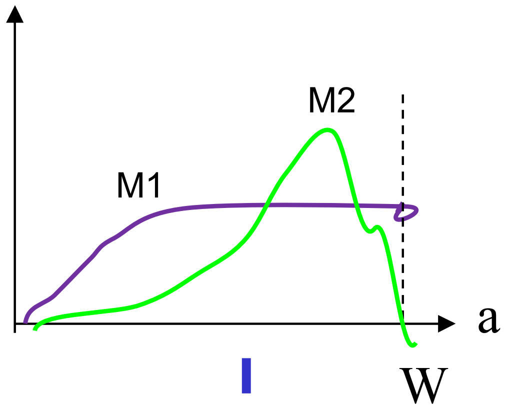
Theoretical vs. experimental ΔK_{th}-data



(data compilation from FKM-Guideline 2001)

Measurement of residual stress profiles by the cut-compliance technique



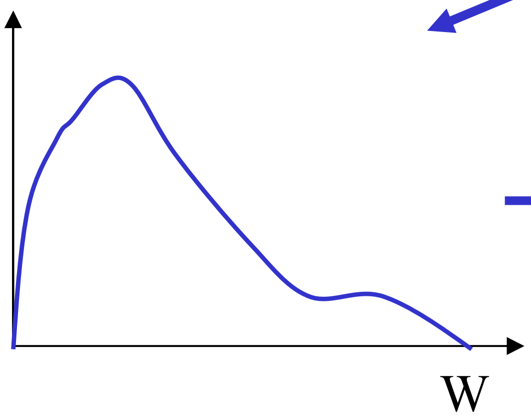


fracture mechanics

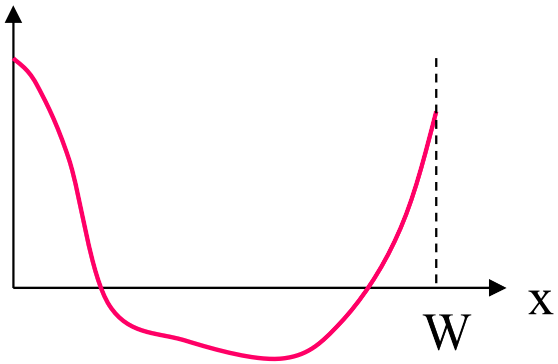
$$K_{Irs} = \frac{E'}{Z(a)} \cdot \frac{d\varepsilon_M}{da}$$

inversion by series expansion of $\sigma_{rs}(x)$

Stress intensity factor K_{Irs}



residual stress σ_{rs}



inverse analysis

$$K_{Irs}(a) = \int_0^a h(x,a) \cdot \sigma_{rs}(x) \cdot dx$$

Z(a): influence function

h(x,a): weight function

Summary and conclusions

- Classical fatigue and FM do not compete with each other, but complement each other .
- Concerning FM, the actual question is not **when**, but **how** to apply FM adequately and efficiently.
- FM indispensable in the following cases:
  - Weldments
  - Sharp notches or undefined notch radii
  - At hot-spots in FEM-analysis
  - To assess NDI-results
  - When using a safe life concept
  - In a failure analysis
- A universal da/dN-Curve seems to exist. Further research is required to determine it.